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Objectives

Derive criteria to establish sail system similarity based on governing equations
Enumerate criteria and available design parameters as guidelines for designing scaled models for testing and demonstration
Demonstrate the use of these criteria and a proposed limit for satisfactory scaling

Procedure:

Select a set of governing equations for the sail, boom and boundary conditions.
Cast the governing equations into dimensionless form
Define the set of similarity criteria and characteristic terms from the non-dimensional equations

Non-Dimensional Sail Model:

Based on a thin plate model for the sail that assumes bending stiffness is negligible with respect to extensional stiffness (Von Karmen plate or Fopple membrane)

Dimensionless Equations:

$$F_{xxxx} + 2P_1 F_{xxyy} + P_1^2 F_{yyyy} = (w_{yy})^2 - w_{xx} w_{yy}$$

$$1 + F_{yy} w_{xx} + F_{xx} w_{yy} - 2F_{xy} w_{xy} = w_{xx}$$

$$F_{yy} = P_2 \text{ } |(N_x) \text{ boundary}$$

$$F_{xx} = P_3 \text{ } |(N_y) \text{ boundary}$$

$$F_{xy} = P_4 \text{ } |(N_{xy}) \text{ boundary}$$

Non-Dimensional Boom Model:

Based on a uniform boom with flexural and axial rigidity

Dimensionless Equations:

$$v_{ssss} - P_{b1} v_{ss} - 1 = P_{b2} v_{tt}$$

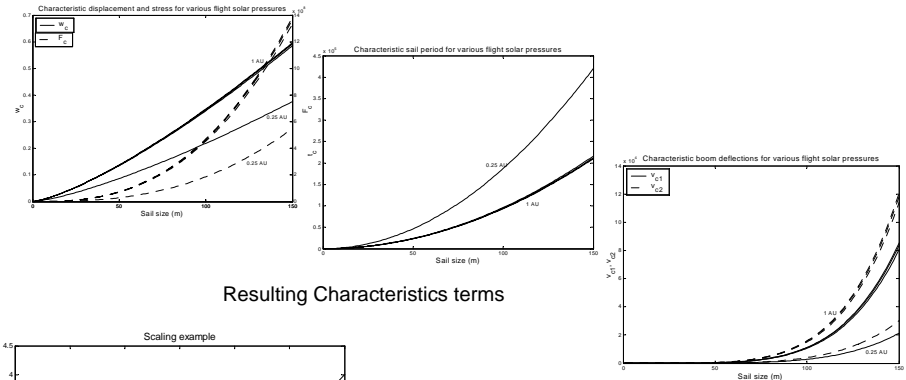
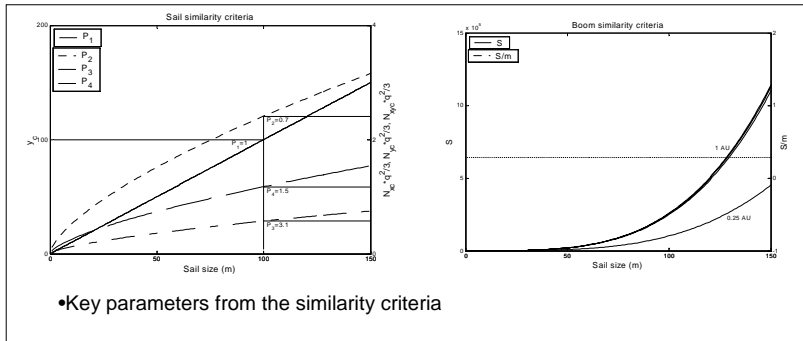
$$v_{ss} = 1$$

Summary of Sail Model Criteria

Criteria	Char. Terms
$P_1 = \frac{x_c}{y_c}$	$w_{xx} = \sqrt{\frac{qk}{hEP_1}}$
$P_2 = N_{xx} \sqrt{Ehq}^{-2} x_c^{-2}$	$F_c = \sqrt{\frac{Eq^{-2} x_c^2}{h^2 P_1^2}}$
$P_3 = N_{yy} \sqrt{Ehq}^{-2} x_c^{-2}$	$t_c = \sqrt{\frac{\rho h x_c}{qP_3}}$
$P_4 = N_{xy} \sqrt{Ehq}^{-2} x_c^{-2}$	
$P_{b1} = \frac{Px}{2S}$	$v_{c,1} = \frac{qx^{-2} P_1}{8S}$
$P_{b2} = \frac{Px}{2S}$	$v_{c,2} = \frac{qx^{-2} P_1}{4\sqrt{2}S}$

¹ Terms exists as a similarity criteria if a prestress is defined in the sail, otherwise results in a characteristic definition of boundary loads
² Boom deflection due to tip load characteristic of ABLE Sail design
³ Boom deflection due to distributed lateral load characteristic of L'Garde Sail design

Results



Scaling Example:

Six criteria with eleven unique variables exist within these criteria, ($x_c, y_c, E, h, q, m, N_{xx}, N_{yy}, N_{xy}, S$ and t).
If, x_c, q, E, h, ρ , are predefined, then six parameters remain ($y_c, N_{xx}, N_{yy}, N_{xy}, S$ and t).
Assume Boundary conditions are not completely met (follow a linear boundary rule rather than that prescribed by P2-P4)
Resulting sail can scale by approximately a factor of 3 to stay within 1 order of magnitude

